

Time: 3 Hours

SECTION - A

Answer any FIVE questions. Each question carries 4 marks.

5QX4M=20M

- 1) Find the equation of the plane through $(4, 4, 0)$ and perpendicular to the planes $x + 2y + 2z = 5$ and $3x + 3y + 2z - 8 = 0$
- 2) Find the distance between the planes $2x - y + 3z = 6$ and $-6x + 3y - 9z = 5$
- 3) Find the point of intersection of the lines $\frac{x-1}{-3} = \frac{y-2}{2} = \frac{z-3}{2}$ and $\frac{x-1}{3} = \frac{y-5}{1} = \frac{z}{-5}$
- 4) Prove that the lines $\frac{x-3}{4} = \frac{y-2}{1} = \frac{z-1}{3}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar
- 5) Find the equation of the sphere of radius 3 concentric with the sphere $x^2 + y^2 + z^2 - 2x - 2y - 2z = 1$
- 6) Find the equation of radical plane of the coaxial system whose limiting points are $(-1, 2, 1)$ and $(-2, 1, -1)$.
- 7) Find the equation to the cone which passes through the three coordinate axes and the lines $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and $\frac{x}{2} = \frac{y}{1} = \frac{z}{1}$
- 8) Find the reciprocal cone of $ax^2 + by^2 + cz^2 = 0$

SECTION-B

Answer ALL questions. Each question carries TEN marks.

5QX10M=50M

- 9) a) A variable plane is at a constant distance p ($p \neq 0$) from the origin. It meets the coordinate axes in A, B, C. Through A, B, C planes are drawn parallel to the coordinate planes. Show that the locus of their point of intersection is $x^{-2} + y^{-2} + z^{-2} = p^{-2}$. (OR)
 b) Prove that the equation $2x^2 - 6y^2 - 12z^2 + 18yz + 2zx + xy = 0$ represent a pair of planes, and find the angle between them.
- 10) a) Find the S.D. between the lines $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ and $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. Hence show that the lines are coplanar. (OR)
 b) Find the perpendicular distance of the point $(-1, 3, 9)$ from $\frac{x-13}{5} = \frac{y+8}{8} = \frac{z-31}{1}$.
- 11) a) Find the equations of the spheres passing through the circle $x^2 + y^2 = 4, z = 0$ and is intersected by the plane $x + 2y + 2z = 0$ in a circle of radius 3. (OR)
 b) Find the pole of the plane $x + 2y + 3z = 7$ w.r.t the sphere $x^2 + y^2 + z^2 - 2x - 4y - 6z + 11 = 0$.
- 12) a) Find the limiting points of the coaxial system defined by the spheres $x^2 + y^2 + z^2 + 4x - 2y + 2z + 6 = 0$ and $x^2 + y^2 + z^2 + 2x - 4y - 2z + 6 = 0$ (OR)
 b) Find the equation of the cone with vertex $(5, 4, 3)$ and $3x^2 + 2y^2 = 6, y + z = 0$ as base.
- 13) a) Find the equation to the right circular cone whose vertex is $(3, 2, 1)$, axis line $\frac{x-3}{4} = \frac{y-2}{1} = \frac{z-1}{3}$ and semi vertical angle 30° . (OR)
 b) Find the equation of the enveloping cylinder of the sphere $x^2 + y^2 + z^2 - 2x + 4y = 1$ having its generators parallel to the line $x = y = z$.

CLUSTER UNIVERSITY :: KURNOOL
U.G.FIRST YEAR II SEMESTER END EXAMINATIONS APRIL 2024
Programme :B.Sc. Subject : MATHEMATICS
Paper Title : THREE DIMENSIONAL ANALYTICAL SOLID GEOMETRY
Time:3hrs **Max Marks:75**

Section - A

Answer any 05 (Five) questions. Each question carries 05 marks

5 x 5 = 25

1. Find the equation to the plane through the points (2, 2, 1), (9, 3, 6) and perpendicular to the plane $2x + 6y + 6z = 9$.
2. Find the image of the point (1, -1, 5) in the plane $3x - 2y - 4z - 14 = 0$.
3. Find the equation of the plane through the origin and containing the line is $x - 3y + 2z + 3 = 0 = 3x - y + 2z - 5$.
4. Find the equation to the sphere through O (0, 0, 0) and making intercepts a, b, c on the axes.
5. Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ and the point of contact.
6. Find the equation to cone which passes through the coordinate axes and the lines $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and $\frac{x}{3} = \frac{y}{-1} = \frac{z}{1}$.
7. Find the equation of the enveloping cone of the sphere $x^2 + y^2 + z^2 + 2x - 2y = 2$ with its vertex at (1, 1, 1).
8. If the axis of a right circular cylinder of radius r is the z-axis then the equation of the cylinder is $x^2 + y^2 = r^2$.

Section - B

Answer Any ALL Questions.

5 x 10 = 50

9. (a) A plane meets the coordinate axes in A, B, C. If the centroid of ΔABC is (a, b, c), then show that the equation of the plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$

OR

- (b) Prove that the equation $2x^2 - 6y^2 - 12z^2 + 18yz + 2zx + xy = 0$ represents a pair of planes, and find the angle between them.

10. (a) Show that the lines $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$; $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$ are coplanar. Also find their point of intersection and the plane containing the lines.

OR

- (b) find the shortest distance and the equation of S.D line between the lines

$$\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}, \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$$

11. (a) A sphere of radius k passes through the origin and meets the axes in A,B,C. Show that the centroid of the triangle ABC lies on the sphere $9(x^2 + y^2 + z^2) = 4k^2$.

OR

- (b) Show that the two circles $x^2 + y^2 + z^2 - y + 2z = 0$, $x - y + z = 2$; $x^2 + y^2 + z^2 + x - 3y + z - 5 = 0$, $2x - y + 4z - 1 = 0$ lie on the same sphere and find its equation.

12. (a) Find the limiting points of coaxial system determined by two spheres whose equations are $x^2 + y^2 + z^2 + 3x - 3y + 6 = 0$ and $x^2 + y^2 + z^2 - 6y - 6z + 6 = 0$

OR

- (b) Find the vertex of the cone $7x^2 + 2y^2 + 2z^2 - 10zx + 10xy + 26x - 2y + 2z - 17 = 0$

13. (a) Prove that the equation $\sqrt{fx} \pm \sqrt{gy} \pm \sqrt{hz} = 0$ represents a cone that touches the coordinate planes and find its reciprocal cone.

OR

- (b). Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and passes through the curve $x^2 + y^2 = 16$, $z = 0$
